

# On the possibility of practically obfuscating programs

## Towards a unified perspective of code protection

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### Abstract

Barak et al. gave a first formalization of obfuscation, describing an obfuscator  $\mathcal{O}$  as an efficient, probabilistic “compiler” that takes in input a program  $P$  and produces a new program  $\mathcal{O}(P)$  that has the same functionality as  $P$  but is unintelligible. This means that any result an obfuscated program can compute is actually computable given only an input/output access (called oracle access) to the program  $P$ : we call such results trivial results. On the basis of this informal definition, they suggest a formal definition of obfuscation based on oracle access to programs and show that no obfuscator can exist according to this definition. They also try to relax the definition and show that, even with a restriction to some common classes of programs, there exists no obfuscator.

In this work, we show that their definition is inaccurate and lacks a fundamental property, that we formalize by the notion of oracle programs. Oracle programs are an abstract notion which basically refers to perfectly obfuscated programs. We suggest a new definition of obfuscation based on these oracle programs and show that such obfuscators do not exist either. Considering the actual implementations of “obfuscators”, we define a new kind of obfuscators,  $\tau$ -obfuscators. These are obfuscators that hide non trivial results at least for time  $\tau$ . By restricting the  $\tau$ -requirement to deobfuscation (that is outputting an intelligible program when fed with an obfuscated program in input), we show that such obfuscators do exist. Practical  $\tau$ -obfuscation methods are presented at the end of this paper: we focus more specifically on code protection techniques in a malware context.

Based on the fact that a malware may fulfill its action in an amount of time which may be far larger than the analysis time of any automated detection program, these obfuscation methods can be considered as efficient enough to greatly thwart automated analysis and put check on any antivirus software.

## 1 Introduction

Obfuscation is the process of making circuits or programs unintelligible. This article will focus more specifically on computer programs. When developing programs, the intelligible source code

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is translated into some binary language, whose structure follows quite precisely the structure of the source code. Thus the initial source code could be retrieved by using some common reverse engineering techniques. Depending on the compiled language being used, this process is more or less long and difficult but is eventually carried out if no protection measure has been taken. Some technologies are particularly sensitive besides, like Java and .NET technologies: the source code is compiled in a very high level byte code that is essentially a line by line binary translation of the source code. Thus, many tools exist for both technologies, that allow decompiling such binary codes in a matter of seconds.

Commercial programs are particularly concerned with this problem, due to the need of protection of the software. Another application of obfuscation is in virology among others. A major concern of malware developers is to make malware undetectable by any sequence-based detection techniques: this is achieved by making the malware polymorphic or metamorphic. Polymorphism/metamorphism is the property of the malware of never replicating in the same way, which means that there is an infinite number of distinct copies of the malware. Obfuscation provides a mean for the malware to achieve or increase its polymorphism since it transforms a program into an obscure program yet producing the same results as the initial program.

This context makes obfuscation interesting to study, at least to know if it might exist or not and maybe to find out some obfuscation methods that might be used by the previous applications. Barak & al. [BGI<sup>+</sup>01] gave in 2001 a first formalization of obfuscation. They propose a formal definition of obfuscators and their main result has been to prove that obfuscators did not exist. They proved that this result still holds for approximate obfuscators, i.e. obfuscators producing programs that are approximately functionally equivalent to the input program. They also considered the case of restricting obfuscators to some specific classes of programs or circuits and showed that, for some very common classes, such obfuscators could not exist either.

This article will aim at providing a theoretical as well as a practical framework for the study of obfuscation methods in the general context of code protection. We will start with a theoretical analysis of obfuscation in Section 3. By going into Barak & al's definition in depth, we point out that there is an inaccuracy. The proof of some of the previous impossibility results is based on this flaw, which makes it quite relevant. We will therefore propose a new definition of obfuscators and show that the impossibility results are still valid for our new definition.

Given these impossibility results, we will suggest relaxed definitions of obfuscators that would allow the concept of obfuscation to be applied in some restricted way. We describe more particularly obfuscators whose efficiency can be limited in time. These  $\tau$ -obfuscators require that the obfuscation remains effective at least for time  $\tau$ .

Obfuscation must be considered furthermore along with its reversing process, that is to say deobfuscation: we will propose a formal definition of deobfuscation in Section 4 and give some basic results and suggest.

We will finally get into more practical details in Sections 5 and 6 by first describing some obfuscation methods that might comply with our relaxed requirements of obfuscation and then describing how a malware could use  $\tau$ -obfuscation for stealth and possibly polymorphism purpose. In this context, we propose a unified approach of code protection in which obfuscation is considered along with other techniques (polymorphism/metamorphism, light code armouring, encryption).

## 2 Definitions and notations

### 2.1 Notations

Let us consider two algorithms  $A$  and  $B$ .

We denote by  $|A|$  the size of  $A$  and by  $t(A(x))$  its running time on input  $x$  (provided that  $A$  halts on input  $x$ ).

We say that  $A$  is  $\mathcal{PPT}$  or that  $A \in \mathcal{PPT}$  if  $A$  is a probabilistic polynomial time algorithm [Pap95, §11.2].

Let us denote by  $A(x) \nearrow$  the fact that the computation of  $A$  on an input  $x$  never halts (and consequently never yields a result).

$A$  and  $B$  are said to be functionally equivalent, which we denote by  $A \equiv B$ , if and only if they produce the same results on their inputs. If  $A$  and  $B$  are defined on  $\mathcal{D}_A$  and  $\mathcal{D}_B$ , then:

$$\forall x \in \mathcal{D}_A, \begin{cases} \mathcal{D}_A = \mathcal{D}_B \\ B(x) \nearrow \text{ if } A(x) \nearrow \\ B(x) = A(x) \text{ otherwise.} \end{cases}$$

Let us consider that  $A$  and  $B$  are probabilistic algorithms. Given two inputs  $x$  and  $y$ , we denote by  $\Pr[A(x)] \approx \Pr[B(y)]$  the fact that the distributions of outputs of  $A$  and  $B$  on their respective inputs  $x$  and  $y$  are the same, up to a negligible difference.

$A(1^t)$  refers to the algorithm  $A$  limited in its running time by time  $t$ :

$$\forall x \in \{0, 1\}^*, A(x, 1^t) = \begin{cases} A(x) & \text{if } A(x) \text{ can be evaluated in less than time } t \\ \perp & \text{otherwise (the function is undefined on input } x) \end{cases}$$

$TC_0$  programs are polynomial programs, which means that their running time is polynomial in the size of the input. If  $P$  is a  $TC_0$  program, then there is a polynomial  $p$  such that:

$$\forall x, t(P(x)) = p(|x|)$$

## 2.2 Functional decomposition

Let  $f : \{0, 1\}^p \rightarrow \{0, 1\}^q$  be a function, with  $p, q > 0$ . The function  $f$  has a unique functional decomposition in  $f_0$  and  $f_1$  that we note:  $f = f_0 \# f_1$ .  $f_0$  and  $f_1$  are defined as follows:

$$f_0 : \begin{cases} \{0, 1\}^{p-1} & \rightarrow \{0, 1\}^q \\ x & \mapsto f(0.x) \end{cases}$$

$$f_1 : \begin{cases} \{0, 1\}^{p-1} & \rightarrow \{0, 1\}^q \\ x & \mapsto f(1.x) \end{cases}$$

where  $0.x$  is the result of the concatenation of 0 and  $x$ .

## 2.3 Obfuscation

We say that we have an oracle access to a program  $P$  when we do not have access to a program's description of  $P$  but, for any input  $x$ , we have access to  $P(x)$  in a polynomial time (in the size of  $P$ ). Oracle access to  $P$  is therefore equivalent to an input/output access to  $P$ . An algorithm  $A$  having an oracle access to  $P$  is denoted by  $A^P$ . Note that the only possible queries are input/output ones since we have no access to a program's description of  $P$ .

Let  $\Omega$  be the set of obfuscators (as we will define them in this article) and let  $\Pi$  be the set of programs that we want to obfuscate using obfuscators of  $\Omega$ .  $\overline{\Omega}$  is the set of deobfuscators<sup>1</sup> and  $\overline{\Omega}_{\mathcal{O}}$  is the set of deobfuscators with regards to a given obfuscator  $\mathcal{O} \in \Omega$ .

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<sup>1</sup>The concept of deobfuscation will be exposed in Section 4.

Let  $R$  be a result on programs of  $\Pi$ . Typically,  $R$  is in fact a property of  $\Pi$  and thus can be defined as a predicate. We say that  $R$  is trivial if it can be computed for a program  $P$  by simply querying  $P$  on some specific inputs.

Thus an obfuscator is expected to hide any non trivial result. In other words, any result an adversary can compute from an obfuscated program is actually trivial. Barak & al. gave the following definition:

**Definition 1.** *A probabilistic algorithm  $\mathcal{O}$  is an obfuscator if it satisfies the following properties:*

- *Functionality: for any  $P \in \Pi$ ,  $P$  and  $\mathcal{O}(P)$  compute the same function.*
- *Polynomial slowdown: for any  $P \in \Pi$ , the running time and size of  $\mathcal{O}(P)$  are at most polynomially larger than the size and running time of  $P$ .*
- *"Virtual black box": for any adversary  $A \in \mathcal{PPT}$ , there is a simulator  $S \in \mathcal{PPT}$  such that:*

$$\forall P \in \Pi, \Pr[A(\mathcal{O}(P))] \approx \Pr[S^P(1^{|P|})]$$

The second condition is rather wide and in practice, we would require that the runtime of the obfuscating program is simply linear in the running time of the source program. Since the slowdown property aims at defining the efficiency of the obfuscator, it is completely up to choice.

From an intuitive point of view, the "Virtual black box" property simply asserts that anything that can be efficiently computed from  $\mathcal{O}(P)$  can also be computed given oracle access to  $P$ . Thus any property that may be computed is necessarily trivial.

Probabilities are taken over the coin tosses of  $\mathcal{O}$ ,  $A$  and  $S$ .

An obfuscator is said to be efficient if it runs in polynomial time (in the size of the input program).

### 3 New formalization and impossibility results

#### 3.1 Barak & al's results

Barak & al proved in particular that obfuscators, as they had formally defined them, did not exist. We will detail the proof of this result later in this section. Let us start with the following strong implication, that was implicitly expressed in Barak et al.'s paper.

**Proposition 1.** *No information can be hidden in a program, using Barak & al.'s obfuscators.*

*Proof.* The proof is a generalization of the proof of the Theorem 4.12, in Barak & al.s' paper.

Barak & al. define among others a set  $\mathcal{H}$  of totally unobfuscatable functions. Let  $f \in \mathcal{H}$  be one of those functions.  $\mathcal{H}$  requires the existence of a result  $\pi$  on  $f$  such that, when given oracle access to  $f$ ,  $\pi(f)$  yields a pseudo-random result when  $f$  is pseudo-randomly chosen in  $\mathcal{H}$ .  $\mathcal{H}$  also requires the existence of an algorithm  $A$  such that, for any program  $Q$  that computes  $f$ , then  $A(Q)$  computes  $f$ . Barak & al show that  $\mathcal{H}$  exists ( $\mathcal{H}$  being infinite).

Let us denote this information by  $R$ .

Now define program  $P$  by :

$$\forall x \in \{0, 1\}^*, P(x) = (f(x), \pi(f) \oplus R)$$

where  $f$  is such an unobfuscatable function.

This program  $P$  returns a couple of values that gives us no information on  $R$  when given oracle access to  $P$  (since oracle access to  $P$  means oracle access to  $f$  and  $\pi(f)$  is pseudo-random

in this case). Yet, for any obfuscation  $\mathcal{O}(P)$  of  $P$ , we have access to a function computing  $f$  (by simply keeping the first value): thus we can compute  $f$  itself, using  $A$ , thereby allowing us to compute  $\pi(f)$ . Eventually, we are able to compute  $R$ .

Since  $\mathcal{H}$  is infinite, there is also an infinity of such programs  $P$ . □

Barak & al. applied this result to private-key encryption schemes among others, proving that there exist such schemes where the key can not be retrieved by oracle access though it can be retrieved when possessing any representation of this (obfuscated) scheme.

This result already illustrates a lack in the previous definition of obfuscators. Let's get into more details with Barak & al's main impossibility result.

The proof relied on the decomposition of an input program  $P$  into programs  $P_1$  and  $P_2$ :  $P = P_1 \# P_2$ . Functionalities of  $P_1$  and  $P_2$  are preserved by any obfuscation, since the functionality of  $P$  is preserved. Let us now consider an adversary  $A$  that computes, on the input program  $P$ ,  $P_1(P_2)$ . In the specific case of  $P_1$  testing a functionality of  $P_2$ , this result is preserved by any obfuscation: for any obfuscator  $\mathcal{O} \in \Omega$ ,  $\Pr[A(P)] = \Pr[A(\mathcal{O}(P))]$ .

We see that, since  $P_1$  tests a functionality of  $P_2$ , it might be difficult to compute this result with only an oracle access to  $P$ . First let's note that oracle access to  $P$  means oracle access to  $P_1$  and  $P_2$ . Now we shall remember that the simulator  $S$  is defined for a given adversary  $A$  but for any program  $P$ : thus we do not previously know what computes neither  $P_1$ , nor  $P_2$ . And we cannot emulate  $P_1$  since we only have an input/output access to it: the only way to compute the result is to compute  $P_1$  on an input. And this input must be a program description of  $P_2$ , to which we also only have an input/output access. Hence, we see that no algorithm  $S$  will be able to simulate  $A$ .

In Barak & al's paper, two random values  $\alpha$  and  $\beta$  are considered:  $P_1$  tests whether  $P_2(\alpha)$  equals  $\beta$ . Since  $\alpha$  and  $\beta$  are not predefined and cannot be guessed by oracle access to  $P$ , no generic simulator can be defined. So, if we consider the case of  $P_2$  such that  $P_2(\alpha) = \beta$ , with  $\mathcal{O}(P) = P_1 \# P_2$ , then  $A(\mathcal{O}(P))$  will always return 1 though  $S$  is unable to compute the result given oracle access to  $P$ .

### 3.2 New formalization

The proof of this impossibility result points out an inaccuracy of the previous definition. Consider that you have in your hands a perfectly obfuscated program, without wondering if there is an obfuscator capable of producing it. Then, should we really be unable to compute the previous result? Although the program is perfectly obfuscated, we have nonetheless a program  $Q$  that is functionally equivalent to  $P$ . So we are able to decompose it into  $Q = Q_1 \# Q_2$  and to compute  $Q_1(Q_2)$ . Thus, whatever the obfuscation, this result cannot be hidden, though the current definition of obfuscators requires it.

What should be taken into account is that an obfuscator does not hide a program's description. We address this lack by defining *oracle programs*. They represent perfectly obfuscated programs, as we just mentioned one in the previous paragraph. More formally, we give the following definition of oracle programs:

**Definition 2.** *For any program  $P \in \Pi$ , we define an oracle program  $\hat{P}$  as a program that is functionally equivalent to  $P$  but that can only be accessed in an oracle way.*

Note that such programs have a purely formal interest: their use does not require any preliminary proof of their existence. There is in particular no equivalence between their existence and the existence of obfuscators. Although the existence of (ideal) obfuscators would give us a mean of producing such programs, being able to produce oracle programs might rely on some

exponential time algorithm or some black box access to an algorithm, which is incompatible with the basic requirements of an obfuscator.

Given a program  $P$  and its associated oracle program  $\dot{P}$ ,  $\dot{P}$  is an oracle program of any program  $Q$  functionally equivalent to  $P$ .  $\dot{P}$  should therefore be seen as an equivalence class containing all programs functionally equivalent to  $P$ , hence the notation.

We should now use oracle programs in the definition of obfuscators, by basically requiring that obfuscated programs behave like oracle programs:

**Definition 3.** *A probabilistic algorithm  $\mathcal{O}$  is an obfuscator if it satisfies the following properties:*

- *Functionality: for any  $P \in \Pi$ ,  $P$  and  $\mathcal{O}(P)$  compute the same function.*
- *Polynomial slowdown: for any  $P \in \Pi$ , the running time and size of  $\mathcal{O}(P)$  are at most polynomially larger than the size and running time of  $P$ .*
- *"Virtual black box": for any adversary  $A \in \mathcal{PPT}$ , there is a simulator  $S \in \mathcal{PPT}$  such that:*

$$\forall P \in \Pi, \Pr[A(\mathcal{O}(P))] \approx \Pr[S(\dot{P}, 1^{|P|})]$$

We intuitively expect that such obfuscators will not exist either: we will show this result later in this section. Thus, we should try to adopt a relaxed definition of obfuscators.

In a first approach, we could require that only some results are hidden. In other words, we should allow some non trivial results to be computed, although requiring other non trivial results to be hidden. But then on what basis should we consider that a result is not important enough to require its concealment? Actually, we could restrict the obfuscation to one requirement: require that no deobfuscation can be performed from the obfuscated program. But this requires a solid formalization of deobfuscation, which is all the more difficult as some results can usually be irreversibly hidden. This will be discussed in section 4.

In a second approach, we could require that the obfuscation remains effective at least for a certain time. In other words, we allow non trivial results to be computed but not in less than a certain time  $\tau$ . We call  $\tau$ -obfuscators, obfuscators with such a requirement.  $\tau$ -obfuscators are defined in the same way as obfuscators except for the virtual black box property:

**Definition 4.** *A probabilistic algorithm  $\mathcal{O}$  is a  $\tau$ -obfuscator if it satisfies the following properties:*

- *Functionality: for any  $P \in \Pi$ ,  $P$  and  $\mathcal{O}(P)$  compute the same function.*
- *Polynomial slowdown: for any  $P \in \Pi$ , the running time and size of  $\mathcal{O}(P)$  are at most polynomially larger than the size and running time of  $P$ .*
- *"Virtual black box": for any adversary  $A \in \mathcal{PPT}$ , there is a simulator  $S \in \mathcal{PPT}$  such that:*

$$\forall P \in \Pi, \Pr[A(\mathcal{O}(P), 1^{\tau \times t(\mathcal{O}(P))})] \approx \Pr[S(\dot{P}, 1^{|P|})]$$

This property states that any result that can be computed in less than time  $\tau \times t(\mathcal{O}(P))$  — where  $t(\mathcal{O}(P))$  is the time needed to obfuscate  $P$  — is actually computable from an oracle program of  $P$ .

We could actually restrict in a first time the  $\tau$ -requirement to deobfuscation. In other words, a  $\tau$ -obfuscator is required to prevent deobfuscation in a time less than  $\tau \times t(\mathcal{O}(P))$ . We will show that, unlike the previous obfuscators, such  $\tau$ -obfuscators do exist. Section 6 will describe techniques that efficiently implement the  $\tau$ -obfuscation concepts. We consider as an open problem to determine whether  $\tau$ -obfuscators exist in the general case (i.e. without restricting the  $\tau$ -requirement to deobfuscation).

### 3.3 Impossibility results

We show that obfuscators do not exist; in other words, there is at least one non trivial result that can be hidden by no probabilistic algorithm.

**Proposition 2.** *Program obfuscators do not exist, as defined with respect to Definition 3.*

*Proof.* Let us first expose some preliminary considerations.

Consider a program  $P$  and an input  $x$ , this program performs several computations to eventually yield an output  $P(x)$ . What we are interested in is the way the program computes this result and thus we are interested in obfuscating these computations: this obfuscation might insert some intermediary steps like decompressing or decrypting some code before executing it. These steps could be performed beforehand and would lead to an executable code that performs the initial computations.

We first consider this executable code only, *i.e.* we consider that it can be retrieved by an algorithm, through a more or less long process.

Code of computer programs consists of simple arithmetic computations, function calls and more complex branches that define conditional jumps or loops. These branches make the global structure of the program.

We show that the global algorithmic structure of an obfuscated program is constrained by its initial algorithmic structure.

Let us consider this structure. Since it is valuable information on the program's algorithm, this structure should be obfuscated as well. Thus an obfuscator should produce a program with an independent structure that still yields the same results. Yet we show that an obfuscator cannot hide this structure and preserve the program's functionality.

In order to produce a fully independant program, an obfuscator will have to be aware of what the initial program computes. Determining this is undecidable.

Let us consider an algorithm performing this abstraction. It would have to read the program in a linear way. When faced with structures that might result in a non termination behaviour, being able to interpret their result would mean being able to decide of their termination. Thus, any representation of these structures would have to consider them in an "atomic" way.

Note that this result still holds when restricting to programs that always halt, since such programs are structurally indistinguishable from programs that do not necessarily halt.

Thus an obfuscator will have to produce a program, based on its linear interpretation of the initial program, hence preserving its global structure. This global structure refers precisely to these structures that are responsible for non termination behaviours, namely the loops (in their semantic meaning).

More specifically, this means that complex computations will always be performed in the same way as they were in the original program. Note that simple computations might of course be obfuscated and some code might be added that complicates that structure. Yet, when faced with loops whose number of iterations is unknown, the obfuscator will have no other choice but to produce a similar loop in the obfuscated program. These very loops are the ones responsible for the non termination of programs : being able to interpret the computations of such loops would mean being able to decide of programs' termination.  $\square$

Thus the proof relies on this more informal result :

**Proposition 3.** *There is no algorithm that can interpret the functionality of every program.*

The reader will probably note that Proposition 3 is somehow a reformulation of Rice's Theorem.

*Proof.* If this were the case, this algorithm would be able to figure out, on an algorithm-independent level of representation, what a program computes and thereby tell if this program halts or not on a certain input. Since the halting problem is undecidable, such an algorithm does not exist.

This result actually comes from the fact that, since an obfuscator must preserve the functionality of the input program, the output program must not halt on inputs where the input program does not halt. And since the obfuscator can not determine where the input program does not halt, it has to reproduce exactly the weaknesses of the input program that would be responsible for a non-halting behaviour. These weaknesses come from the complex structures of the input program, like infinite loops. Thus this loop structure – considering only loops whose number of iterations is not known in advance – must be preserved as it is.

Consider for instance a loop, whose number of iterations cannot be predicted, then an obfuscator cannot tell whether this loop will always exit at runtime: it will therefore have to produce a loop which performs the same computations and might never exit.  $\square$

Note that this proposition too still holds when restricting to programs that always halt (indistinguishability of programs).

This result rules out a serious lead since ideal obfuscation would be able to produce some random independent program that computes in a different and obscure way the same results. We know that we will not be able to modify the way results are computed but that we can try to obfuscate sequences of simple computations and to insert code that makes the global algorithmic structure look more confused, though it remained untouched. Section 5 will present some methods achieving this, while usual obfuscation methods already exist, that are described in [CTL97].

## 4 Deobfuscation

When trying to define new obfuscation methods, the first consideration is about reversing these methods. This reversing process is called deobfuscation. Thus, when defining a formal framework for obfuscation, it is also interesting to try to define one for deobfuscation as well.

Deobfuscation can be apprehended as the transformation of some obscure program into a more intelligible program, which is functionally equivalent to the input program. More formally, this means producing a new program on which results that were hard to compute on the initial program become easier (in terms of complexity) to compute on this new program. In other words, non trivial results that were hidden by the obfuscation must be computable from the deobfuscated program. The size and running time of the deobfuscated program are necessarily polynomial in the input size and running time, otherwise it would not be intelligible.

As for efficient obfuscators, we say that a deobfuscator is efficient if it runs in polynomial time in the input program’s size. Then, if obfuscators existed, in their purest definition, we could show unsurprisingly that deobfuscators cannot be efficient. Otherwise, if you consider a result that is revealed by the deobfuscation, it could have been computed in polynomial time from the obfuscated program by first deobfuscating the program.

To formally define deobfuscation, we state that a deobfuscator must satisfy an *intelligibility* property. Thus, on the basis of Barak & al’s definition, we say that:

**Definition 5.** *A probabilistic algorithm  $D$  is a deobfuscator if it satisfies both following properties:*

- *Functionnality:*

$$\forall \mathcal{O} \in \Omega, \forall P \in \Pi, D(\mathcal{O}(P)) \equiv P$$



- *Intelligibility:*

$$\forall \mathcal{O} \in \Omega, \forall R \in \mathcal{PPT}, \\ \exists T \in \mathcal{PPT} \text{ s.t. } t(T) = O(t(R)) \text{ and } (\forall P \in \Pi, \Pr [T(D(\mathcal{O}(P)))] \approx \Pr [R(P)])$$

Probabilities are equal up to a negligible function  $\mu(|P|)$ .

Note that a deobfuscator can be defined relatively to a given obfuscator  $\mathcal{O}$ , in which case we would denote this deobfuscator by  $\overline{\mathcal{O}}$ .

More informally, the intelligibility property states that *any non trivial result that can be computed from the unobfuscated program  $P$  can also be computed in approximately the same time from the deobfuscation of any obfuscation of  $P$ .*

As we suggested it, we show that:

**Proposition 4.** *Efficient deobfuscators do not exist, with respect to our definition or to Barak & al.'s definition of obfuscators.*

*Proof.* Let us consider an obfuscator  $\mathcal{O} \in \Omega$  and a deobfuscator  $\overline{\mathcal{O}} \in \overline{\Omega}_{\mathcal{O}}$ . Let  $R$  be a non trivial result on programs  $P$  of  $\Pi$ .

From the definition of deobfuscators, we infer the existence of an algorithm  $T$  computing the same result on  $\overline{\mathcal{O}}(\mathcal{O}(P))$ . Let  $A$  be an adversary that computes, for an input obfuscated program  $\mathcal{O}(P)$ :  $A(\mathcal{O}(P)) = T(\overline{\mathcal{O}}(\mathcal{O}(P))) = R(P)$ .

Then, from the definition of obfuscators, we deduce that  $A$  computes a trivial result of  $\mathcal{O}(P)$ , which means that  $R(P)$  itself can be trivially computed<sup>2</sup>, which contradicts the hypothesis that  $R$  is a non trivial result.  $\square$

This definition of deobfuscators is not actually accurate enough. It does not take into account the existence of irreversible obfuscation methods. A common obfuscation method for instance consists in scrambling the names used in programs — since these names give information on the structure of the code: such a method is irreversible. This question remains open and might be subject to further research.

## 5 Practical obfuscation methods

The results that were exposed up to now allowed us to exclude some leads from the research of obfuscation algorithms. Our impossibility result showed in particular that an obfuscator could not modify the global algorithmic structure of a complex program.

### 5.1 Program's representation

A computer program is represented as a graph, with one initial state and one or more final states. Each vertex is a simple instruction or a conditional branch: unconditional jumps are eliminated since connecting a vertex to an unconditional branch is equivalent to connecting the vertex to the target of this branch.

Then we define aggregates of simple instructions by gathering together successive simple instructions (edge contraction), whose first one only can be the target of a branch. Such simple instruction aggregates will be denoted by the acronym SIA. There is an optimal structuration

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<sup>2</sup>Note that, in the case of Barak & al.'s definition, oracle access to  $P$  is equivalent to oracle access to  $O(P)$ . In the case of oracle programs, we already noticed that two functionally equivalent programs share the same oracle program (hence the notation).

of the program into SIAs and conditional branches: we will work on the basis of this optimal structuration<sup>3</sup>

It is worth noticing that a *reachable* instruction (from any other instruction in the graph) does not necessarily mean it is an *executable* instruction. The actual execution flow may never pass through a given instruction. This point is essential as far as Proposition 6 is concerned.

Figure 1 gives an example of the graph representation of a program. Notice, among the miscellaneous used symbols, the conditional branches with one or more inputs and two outputs: we might write the condition they test between these two outputs.

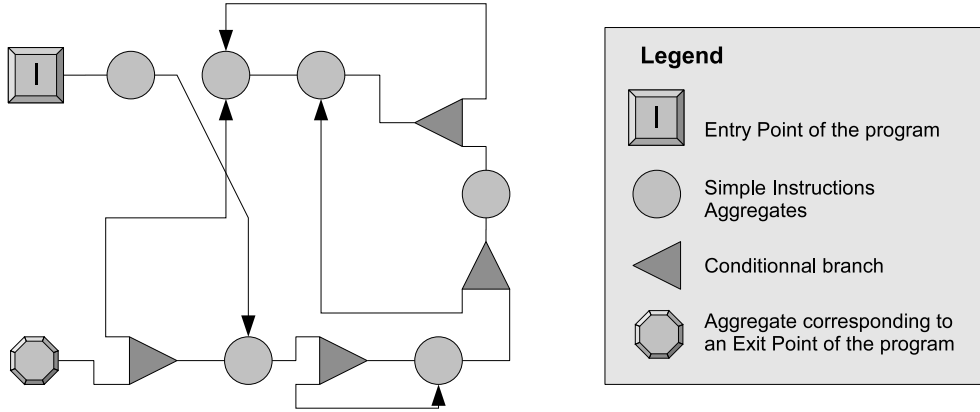


Figure 1: Example of a graph representation of a program

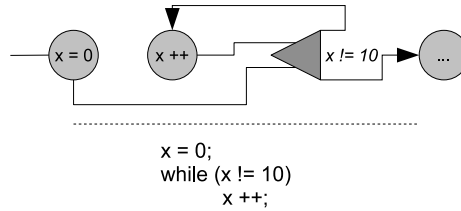


Figure 2: Sample code representation

The last step in extracting the structure of the program is to identify the loop structures. This will allow us to determine the skeleton of the program. We can afterwards test whether a loop has a fixed number of iterations or a variable one: in the second case, we will have to keep the same structure for this loop (except for some basic obfuscation methods that might split the loop into several loops or else). Some of these loops might overlap or interlock: the diagram in figure 1 contains several loops, some of them overlapping or being interlocked. To determine this loop structure, we can use the following algorithm, whose complexity is in  $O(|P|)$  for a program  $P$ .

<sup>3</sup>Such a representation is similar to control flow graphs or flowchart techniques.

We start from the entry point of the program and we explore all possible paths. If a path crosses back one of its current vertices, then we have identified a loop. If a path crosses a vertex that was already explored, then we stop the exploration of this particular path: indeed, if there was to be a loop going through this vertex, then the exploration of this vertex will or would have already identified it. There will be consequently at most  $|P|$  visited vertices, hence a complexity in  $O(|P|)^4$ .

## 5.2 Obfuscation options

The most basic targets of obfuscation are SIAs, as well as loops with a fixed number of iterations. We will from now on consider only loops whose number of iterations is not fixed and previously known.

Whenever such a loop is reached, the obfuscated program has to contain a loop, that performs at least the computations of the initial loop. If an SIA precedes this loop and if its execution is independent of the loop execution, then it can be moved in such a way that it is executed during the execution of the loop — but not after the loop, since we do not know if the loop will terminate. The same is true of instructions that follow the loop, since they are to be executed only when the loop terminates. However, we can consider the case of programs that always terminate: successive blocks with independent execution can then be mixed up. Note that we can also split up the SIAs into several SIAs in order to have more freedom in our obfuscation options.

Whenever two loops overlap in the initial program, they might be executed alternatively. Thus we can choose between two approaches. Either we keep the overlapping, or we separate the two loops, by replicating the common instructions block and updating the branching instructions they contain: if a branch targets a common instruction, then it should be updated to target the equivalent replicated instruction in the current loop. In the first case, the obfuscation of both loops must be carried out simultaneously, while in the second case we can obfuscate in separate ways the two common instructions blocks.

Single loops can also be obfuscated by splitting a loop into several partial loops and obfuscating each partial loop in a different way. The successive execution of all partial loops would be equivalent to the execution of the unobfuscated loop, though being different from one another. We could also insert new instructions between those loops and cancel the effects of these instructions afterwards — provided that we are certain that these cancellation instructions will be really reached.

## 5.3 Obfuscation methods - Targeted obfuscation

There are basically two approaches for obfuscation. Either you plan to puzzle a deobfuscator, or you plan to puzzle an analyst (who might have executed a deobfuscator prior to beginning analysis). Both approaches will rely on two types of obfuscation methods: reversible and irreversible methods.

Irreversible methods are quite a common use in current obfuscators. They can modify names of functions or variables, replace function calls by the function code itself, etc. Hash functions can also be used to implement such irreversible methods: if you store the hash value of a password, then you will still be able to check up the validity of a password without revealing it. We can use this method in comparisons to constant values for instance<sup>5</sup>.

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<sup>4</sup>Actually,  $|P|$  refers here to the number of states in the graph representation of this program, knowing furthermore that some of these states are simple instructions aggregates.

<sup>5</sup>Note that using hash functions means that there is a risk of collision, since a hash function is not injective.

Reversible methods refer to methods such that we can work out an intelligible, yet equivalent, code. Note that we do not require of such obfuscation methods that their effects be cancelled and the initial code restored: this would be meaningless since you can always slightly change the initial code without changing the obfuscated code. Obfuscation methods that were described in section 5.2 — which consisted in the reorganization of the loops structure — are usually reversible and we are therefore interested in the time needed to restore the loops structure after obfuscation.

The following sections will describe several obfuscation methods. We first think of obfuscation with code insertion. This consists in the insertion of useless code into the program. It can either be dead code, which means code that is never to be executed, or it can be code that can actually be executed but whose effects are void or cancelled afterwards. We also describe the obfuscation of the basic structure of programs, and more specifically methods to obfuscate boolean tests. After the use of such methods, the output program should be once more obfuscated to conceal as much as possible the use of these methods (since they might insert some predefined template sequences of instructions).

### 5.3.1 Obfuscation with dead code insertion

Dead code insertion can be achieved using any valid code. The insertion point must then be coherently chosen, since a deobfuscator will easily eliminate any code that cannot be reached from the entry point of the program. Dead code cannot therefore be inserted after an unconditional branch (`goto`) or a return instruction (`return`) without any branch pointing to the code: if there is a branch pointing to this code, the branch must also be accessible from the entry point and so on. Actually, a deobfuscator will basically keep all paths that start from the entry point of the program. However, if we consider that code and data are mixed up and cannot be distinguished, this code will not be deleted, though it will not appear either in any generated source code, which makes this insertion useless.

Dead code has to be reachable though it must never be executed. We will therefore rely on the following results, to implement code insertion:

**Proposition 5.** *The problem of knowing whether a piece of code is executed, for a given input, is undecidable.*

*Proof.* This problem is actually a reformulation of the halting problem.

Let us proceed by reductio ab absurdum. We assume that this problem is decidable. Thus there exists an algorithm  $A$  that can tell whether, for a given input  $x$ , a piece of code is reached during execution of a program  $P$ .

Then applying  $A$  to every return instruction of  $P$  enables us to decide whether execution of  $P$  terminates on input  $x$ . Hence a contradiction.  $\square$

Now we can establish a more general result.

**Proposition 6.** *The problem of knowing whether a piece of code is ever executed is undecidable.*

In other words, there is no algorithm (whatever its complexity) that can tell with certainty if a piece of code is dead code or not.

---

Thus, there is a very low probability that the functional equivalence is not fulfilled. However, let us consider the case of a loop on a variable that is incremented from 0 to  $N$ . If  $N$  is small, then the functional equivalence will be met with probability 1, whereas a larger  $N$  will make the functional equivalence fail with probability 1.

*Proof.* We use the previous result.

Given a program  $P$  and an input  $x$ , we consider the equivalent program  $Q$  that is the specialization of  $P$  for its input  $x$ . Such a program  $Q$  can be seen as the program  $P$  where the input parameter was turned into a variable initialized to  $x$ . The existence of  $Q$  is proved by Keene's iteration theorem (s-m-n theorem [Kle38]).

Applying proposition 5 to the program  $Q$  proves the result.

Note that this proposition could have been proven by simply using Cantor's diagonal method. Suppose that there exists an algorithm  $A$  deciding our problem. Then define program  $P$  and include in it a piece of code that is executed only if  $A(P)$  returns false. Then if  $A(P)$  returns true, we never execute the piece of code and it should therefore return false. However if it returns false, we execute the piece of code, hence the contradiction.  $\square$

Yet, obfuscation using dead code is not so easy because obfuscators will use predefined techniques to insert dead code: a deobfuscator might try to locate some of these techniques into the code in order to identify dead code. And, if not complex, such a method will be easily reversed by an analyst.

Now let us consider such a dead code. Since it must never be executed but yet has to be reachable, it will have to be the target of a conditional branch that always fails. Thus, the less trivial the condition will be, the more undetectable the dead code will be. The condition must test the value of some program variable, otherwise an obfuscator would only need to run the test once to identify the dead code. So, to locate dead code, a deobfuscator will have to either rely on the detection of known methods, or to be aware of the used variables' values when reaching the condition. The second technique will be made harder to use depending on the way the variables are assigned (this must be complex and scattered among the code), whereas the first technique might be harder to detect if the condition is furthermore complex — possibly randomly generated.

We will use simple conditions that will test the value of a program variable  $x$ . Prior to this test, we initialize  $x$  to a random value, by using a loop with a random number of iterations. A deobfuscator must not be able to tell that the loop has actually a fixed number of iterations. To achieve this, we use hash functions to compare the value of a variable to the expected value: this allows us to hide this expected value. Thus, we propose the following code template:

```
h is a hash function.  
Choose N randomly in a reasonable range.  
Compute A = h(N) and B = h(2 * N).
```

Write in the obfuscated program:

```
int x = 0;  
for (int i = 0; h(i) != A; i++) {  
    x += 2;  
}  
if (h(x) != B) {  
    /* execute dead code */  
}
```

When the loop exits,  $x$  contains the value  $2 * N$  and thus the test always fails. Since  $N$  is not stored in clear and since a deobfuscator cannot trivially guess the number of iterations of the loop, an external observer will either assume that  $h(x)$  really equals  $B$  or will have to execute

the loop<sup>6</sup>. This solution is actually quite weak since, once the loop has been executed, we can determine with certainty whether the code is dead or not. This is due to the fact that a complex enough observer will notice that  $x$  and  $i$  are always initialized to the same value, just before the loop.

Then a new solution would be to initialize  $x$  and  $i$  at the beginning of the program and to possibly use in the obfuscated code the current values of  $x$  or  $i$ , i.e. without resetting them. But  $x$  and  $i$  will keep getting larger and a time will come when no loop will ever execute. To avoid this, we can add a conditional branch (on  $x$  and  $i$ ) inside the loops and target it to a set of instructions resetting  $x$  and  $i$  to random values. The loop might thereby be partially executed, depending on the current values of  $x$  and  $i$ : as a result, these values will not be known in advance.

Thus, it will be a lot harder for a deobfuscator to prove that, when the loop terminates,  $x$  has always the same value which prevents the dead code to be executed. Besides, this solution reduces the performance impact since loops might be partially executed.

Here is a new code template for this obfuscation method:

```
Compute C = h(j), where j is randomly chosen in [0..N-1].
x and i are initialized at the beginning of the program (or function).
```

```
for (; h(i) != A; i++) {
    if (h(i) == C)
        goto lInsertionPoint;
    x += 2;
}
if (h(x) != B) {
    /* Set x and i to random values: */
    x = rand(); i = rand();

    /* Execute the dead code. */

    goto lInsertionPoint;
}
/* Reset x and i:
 * x = i = 0;
 */
lInsertionPoint: /* Resume initial code's execution */
```

A further method to deter an obfuscator from running the loop (with for instance the initial values of  $x$  and  $i$ , which are 0) could use a dependency of the number of iterations in the value of  $x$ . But a weakness will nonetheless remain, since a deobfuscator that would have identified the trick can always run the loop with  $x$  and  $i$  reset to their initial values. Then introducing a dependency in  $x$  would be of no interest, in the purpose of making  $x$  and  $i$  pseudo-unpredictable.

Note that an obfuscation of the final test can also be achieved, to make the dead code more undetectable. Such methods will be described in the upcoming paragraphs. The code can also be obfuscated with other techniques described in this document or in the literature about obfuscation (see [CTL97] for instance).

---

<sup>6</sup> However, there is still a small risk that  $h(i)$  equal  $A$  for  $i \neq N$  and thus that the dead code block be executed. This risk can be ignored if we authorize the obfuscator to generate an invalid program with small probability (which could be indeed allowed in the case of viruses for instance).

### 5.3.2 Obfuscation with void code insertion

Obfuscation with code insertion can also consist in the insertion of code which has actually no effect on the program's behaviour. We call such a code void code. Unlike dead code, void code is really executed.

Void code must also be undetectable, meaning that a deobfuscator should not be able to tell that this code is useless. However, our main advantage is that we know the location of this useless code, while a deobfuscator has no clue about where to look for void code. This makes void code insertion quite powerful.

Void code is implemented in the same way in Turing machines or in computer programs.

In the case of Turing machines, for instance, the read/write head must move on the tape without modifying it or by cancelling the modifications afterwards and it must eventually come back to its initial position. However, this code can modify the tape if it is certain that the modification will have no impact on the expected behaviour of the program. The program might access the modified locations of the tape afterwards, provided that we control its behaviour (for instance, if the code using it was inserted by ourselves).

Similarly, in the case of computer programs (assembler programs), modifications due to void code must have no impact on the further execution of the program. These modifications can affect the memory or registers. In the case of memory, an obfuscator cannot predict if a memory location is used by the program or not: the memory modified by void code must therefore have been explicitly allocated by some inserted code. In the case of registers, the registers must not be used anywhere else than in obfuscator-specific code. Any other modification must be cancelled by the void code.

To be more powerful, the void code should be splitted up into several chunks that are scattered in the code. This would make the detection process much harder, which is all the more interesting as such methods would certainly use predefined void code templates.

### 5.3.3 Obfuscation with code duplication

We can also think of duplicating existing code and obfuscating each duplicate in a different way. Then, we insert a test which leads to the execution of any of these duplicates. We actually have a great deal of latitude for the test, since, whatever the result, the initial code will be executed as expected.

### 5.3.4 Obfuscation with code insertion: conclusion

These methods insert code that often match some predefined template. Thus, as we told before, the inserted code should be obfuscated afterwards with other techniques, to conceal it more efficiently. Inserting code means also choosing specific locations where to insert it. This choice should not be made at random since, for instance, if the code is inserted in deep loops, then the performance impact will not be tolerable.

Yet some positions might be more propitious for code insertion. For instance, when a loop is splitted up into several parts, code could be inserted between these parts to make the detection of the splitted loop harder. Also, when some duplication process is used, inserting code in the duplicates can help in making the duplicates harder to detect. This could be used for instance when replacing function calls by a direct execution of the function body.

Finally, this obfuscation method should be applied before any other method, to help its concealment. It should also be used with other obfuscation processes (like the ones duplicating code).

### 5.3.5 Obfuscation of the conditional structure

The graph representation of programs use SIAs and conditional branches. These conditional branches provide relevant information on the program's behaviour and it would be interested to obfuscate them as well. We describe two ways to achieve this. We could either obfuscate the condition itself, i.e. make it complex enough to be hardly intelligible. The performance impact would be likely to be slight, but this obfuscation would not be very strong. We could also hide which code is executed regarding which value of the condition. This means that the program would only be able to tell at runtime which code is to be executed if the condition is true.

We use therefore the code insertion methods' ideas and propose the following code template:

```
h is a hash function.
<cond> is the tested condition.
If <cond> is true, then <block1> is executed, otherwise <block2> is executed.
Choose N randomly in a reasonable range.
Choose alpha randomly in {0, 1}.
Compute A = h(N), B = h(2 * N + alpha),
C = h(j), where j is randomly chosen in [0..N-1], and D = h(2 * j + alpha).
i and x are initialized to 0 at the beginning (of the program or the function).
```

Write in the obfuscated program:

```
int comparand = B;
if (<cond>) x ++;
for (; h(i) != A; i ++) {
    if (h(i) == C) {
        comparand = D;
        break;
    }
    x += 2;
}
if (h(x) == comparand) {
    if (<cond>) x --;
    /* Insert <block2> if alpha = 0, <block1> otherwise */
} else {
    if (<cond>) x --;
    /* Insert <block1> if alpha = 0, <block2> otherwise */
}
```

When exiting the loop,  $x$  equals  $2 * N$  (resp.  $2 * j$  in case of a partial loop) if the condition is false,  $2 * N + 1$  (resp.  $2 * j + 1$ ) otherwise. Then, according to the value of  $\alpha$  and to the value of the condition, the expected block is executed.  $N$  remains hidden from an external observer, as well as the number of iterations of the loop, since the initial value of  $i$  is unknown. A deobfuscator could not thereby guess the value of  $x$  after the execution of the loop and thus we do not know which block is executed according to which value of the condition. Note however that a deobfuscator could detect this obfuscation, set  $x$  and  $i$  to their initial values, i.e. 0, and execute the loop. The remark in footnote 6 also remains valid.

$x$  and  $i$  could also be reset, as in the examples of code insertion. And rather than resetting them to 0,  $i$  could be first reset to a random value and then  $x$  would be consequently reset (to  $2 * i$  in our case).

A problem of this method would be its impact on the performances, especially when the condition is a loop condition or is inside a loop. This impact depends on the value of  $N$  and



on the reset values of  $x$  and  $i$ . What is more, some tests are not really interesting to obfuscate. For instance, programs often test whether a variable equals a specific value (an error or success value, usually `null` or `0`) and, if so, exit the function or the program. Loop conditions consisting in the comparison of the loop variable to some fixed value are not interesting as well.

Then the first proposed solution, that is to say obfuscating the condition itself, would be more interesting. We could actually apply both methods when obfuscating tests.

To obfuscate conditions, we could in a first time replace the read variables by the expression they were previously assigned to, when possible. This solution is not always possible since this expression could depend on variables that were modified afterwards.

Another very simple method for obfuscating a condition or any expression would consist in the use of known mathematical results that are not trivial. The problem of this approach is that it would not deceive a human observer, provided that he has the required mathematical knowledge. Another problem is its limitation to a restricted set of predefined results, since they have to be previously known. For instance, we could use results on the exponential function or (equivalently) on the power function. The exponential function has the advantage of being bijective, continuous, monotone and defined on  $\mathbf{R}$ : thus we can use this function in any test (for instance:  $(a < b) \Leftrightarrow (\exp(a) < \exp(b))$ ). We can also use the result  $\exp(a) \times \exp(b) = \exp(a + b)$ . We could use this in other applications of tests' obfuscation. In our dead code insertion method, we could use a test of  $\exp(a) \times \exp(b) = \exp(a + b)$ , with any memory values for  $a$  and  $b$  and then execute the dead code if this condition fails: since  $a$  and  $b$  are randomly selected, the result will be unpredictable for a deobfuscator, unless it knows this very property. We can also use such results in the loops that were used by the previous methods, so that their number of iterations becomes undetermined. In the case of a deobfuscator which is unaware of the used results, this method allows us to deceive it with certainty.

### 5.3.6 Generic approach

Let's modelize the program's behaviour, in the specific case of common computer programs — like x86 assembler programs or intermediate language programs (like Java's bytecode or .NET's Common Intermediate Language). We first consider SIAs (Simple Instructions Aggregates). An SIA is a finite sequence of instructions that contains no branch or return instruction. A given SIA starts from an initial state of the memory (basically registers and RAM) and leads after execution to a new state of the memory, depending on the initial state. This new state is precisely described by the set of instructions (as represented in the initial program): since each instruction carries out an operation onto the memory, we can free ourselves from the form of these instructions by representing an SIA as a sequence of memory operations and more specifically as a sequence of writing operations into the memory. We denote such a writing operation by:

```
write{M[i]}: expr{M}
```

$M$  is the memory and  $M[i]$  is a memory location where the data are written. The result is computed from the initial state of the memory, i.e. when entering the SIA. `expr{M}` is an arithmetic expression, that depends on some memory locations and can make function calls: computing this expression yields the data to write into  $M[i]$ . When adding a new `write` operation afterwards, we replace in it all calls to  $M[i]$  by this arithmetic expression. Thus, any SIA can be represented by a sequence of such `write` operations. Note also that the memory is not modified until the end of the execution of the SIA: the operations read the memory in the state it was when entering the SIA. We could actually imagine that all these operations are executed in parallel.

This set of `write` instructions also define a *read memory* (memory locations that are read by the previous arithmetic expressions) and a *written memory* (targets of these `write` operations).

This representation is accompanied with a loss of information: if a variable was modified and then another variable modified according to the new value of the previous variable, this information is lost. But this is all the more interesting as, in the framework of obfuscation, these operations become independent and can thereby be mixed up together. The expressions will also be easier to obfuscate since they might be more complex and thereby be more suitable for obfuscation.

Yet we may note two difficulties, regarding the reverse translation, that is generating an obfuscated code from this more abstract representation. The first one regards the function calls: if, for instance, a constructor of a class is called, calling it several times will produce different instances or even raise errors, though it was initially to be called only once. Such critical function calls must be isolated into different SIAs (the SIA they will belong to must not call them more than once). The second difficulty regards the memory that is read by the operations: as we said, they use the initial memory state and that is why they should be executed in parallel. But, when translating this representation, the operations will be nonetheless executed in sequence, thus modifying the memory that will be read by the following operations. We could address this issue by first computing all data to be written and then write it to the memory.

We can go further and consider the case of conditional branches, as long as we do not enter or exit a loop. When reaching a conditional branch, two paths are defined. Those paths might have already gone through other conditional branches. Thus, for each branch, we add this condition to the representation of the path. A path can eventually be represented by:

```
cond 1: expr{M}
...
cond n: expr{M}
write{M[i_1]}: expr{M}
...
write{M[i_m]}: expr{M}
```

This representation might not be used anymore for generating a new obfuscated code. However, we can still define a read memory and a written memory and then apply reduction rules — as some mathematical tools already do it. This approach can thereby identify which read memory locations are actually useless. Note that the obfuscated code will have to modify at least the initial written memory, with possibly a bijection of the written memory locations.

This same level of representation will be achievable by a deobfuscator and the obfuscation methods must therefore resist it. Thus we should aim at producing expressions that are as complex as possible but also at adding new dependencies in the read memory, i.e. by widening the read memory. These new dependencies should not necessarily be fake. The obfuscation methods we described previously did so when declaring the  $x$  and  $i$  variables in a global scope. But in this case a deobfuscator could choose to go back in the program to detect where the variables might have been set. Introducing loops with unknown or variable number of iterations can thereby dissuade a deobfuscator to do so.

This approach can be implemented by both an obfuscator and a deobfuscator. Therefore it defines new requirements for the obfuscation methods. However, we could make this approach harder to use for a deobfuscator by adding several useless conditional branches in the code. These branches would always fail and thus would not modify the functionality of the code. We could therefore make them target any location of the code, allowing us to create fake loops that would puzzle a deobfuscator, yet having a negligible impact on the running time of the program.

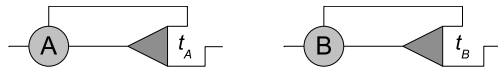


Figure 3: Code before obfuscation

## 5.4 Obfuscation methods - Structural obfuscation

The obfuscation methods we described up to now were targeted, that is that they modify the code in some specific locations, they add new code, make the existing code more obscure, but they always remain focused on a specific part of the program. The approach described in this section consists in an obfuscation of larger pieces of code by a modification of the code's structure. The execution's main steps will still be executed in the same way since they cannot be modified. We therefore describe two different approaches. The first one tries to make the loop structuration as obscure as possible. The second one tries to mix the code in such a way that a deobfuscator cannot detect it but yet the executed code remains unchanged.

### 5.4.1 Obfuscating loops

In the initial program, loops are usually localized, restricted to a small set of instructions, and they are either interlocked or succeeding one another. What we would therefore like to achieve is that the loops stretch on the whole program, without modifying the execution of the program. To do so, we will work on the graph representation of programs. We will nonetheless reuse methods that were defined for targeted obfuscation.

We first consider two loops that succeed one another. We would like to create a single non trivial loop such that a deobfuscator could not tell that this loop is actually made of two distinct loops that execute independently. Figure 3 gives a representation of a program before applying obfuscation methods. The two loops are represented by their bodies  $A$  and  $B$  and their test conditions  $t_A$  and  $t_B$ .

To obfuscate these loops, we use tests in such a way that their result is as much unpredictable as possible. We define a variable  $x$  that we set to an unpredictable value (using the methods that were previously described, like using a loop, mathematical results, and others). The obfuscator must know the current value of  $x$ . Thus we use tests that compare the hash value of  $x$  to a constant value: the results of these tests are known before execution, though a deobfuscator will not be able to tell them.

We now split the loops' bodies into two parts:  $A_1$  and  $A_2$ , and  $B_1$  and  $B_2$ . Finally we modify the code structure according to one method of the figure 4.

In the first method, test  $t$  selects the currently running loop: it compares the hash value of  $x$  to a constant value and thus tells whether we are currently executing loop  $A$  or  $B$ . In the second method, test  $t$  always selects  $A_2$  and test  $t'$  always selects  $B_2$ . Tests  $t_A$  and  $t_B$  are the initial tests and were possibly obfuscated using other methods. However, we will have to either use specific  $x$  variables for each such obfuscation or we will have to ensure that the  $x$  variable is not modified during execution of  $A_1$  and  $A_2$  (resp.  $B_1$  and  $B_2$ ).

Both methods can be generalized to obfuscate several loops at once: in the first method, you simply have to replace the  $t$  test by a sequence of similar tests (this corresponds to a `switch` instruction), and in the second method, the obfuscation should be applied in a circular way ( $A_1$  is connected to  $A_2$  and  $B_2$ ,  $B_1$  to  $B_2$  and  $C_2$ , and so on).

This method can actually be applied to other loops, even when they do not succeed one another.

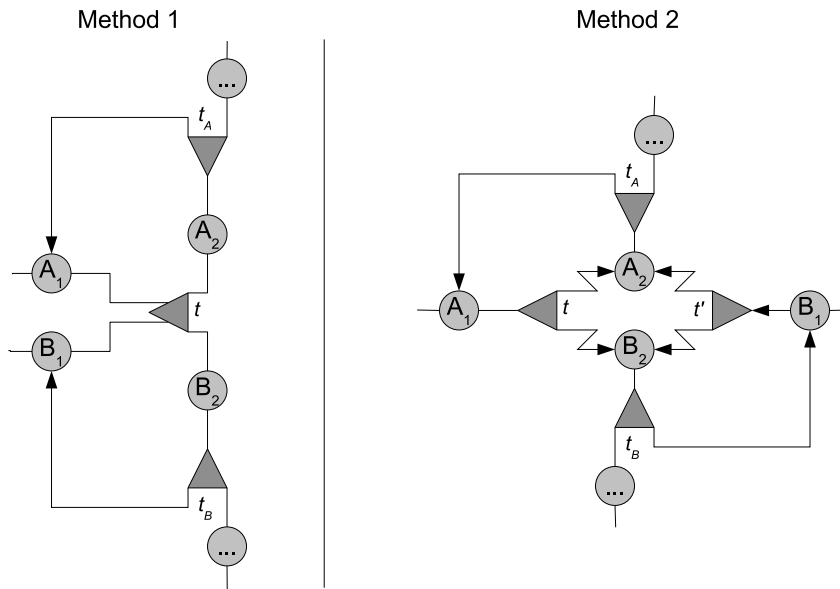


Figure 4: Code after obfuscation

We could also create new fake loops, for instance by taking a piece of code, defining a variable to a random value and creating the following loop:

```
int x = 0;
/* Initialize x to a random value (known by the obfuscator). */
for( ; h(x) != A; x++) {          /* A is the hash of the value of x. */
    /* Execute the piece of code. */
}
```

We could also split up SIAs and create fake branches to the new SIAs in order to make the program's structure more complex.

#### 5.4.2 Code mixing obfuscation

Code mixing obfuscation aims at jumbling up the code in such a way that, thanks to convenient jumps, the code is eventually executed as it was at the beginning, but its structure has been fully modified and is incomprehensible.

We first split the code into several chunks of random size. Then we jumble up these instructions blocks and we connect them one another using branch instructions. If these branches were unconditional, then the method would be trivial and the simple representation of the program as a graph would yield the initial code in a short time.

We therefore use non trivial conditional branches, though all tests will have the same result. Once again, the methods we described in targeted obfuscation fit our needs. We simply have to use hardly reducible expressions as well as a counter  $x$  whose value changes as the program is executed and whose hash value is compared in each test to a constant value.

However, a problem remains since the  $x$  value must change as the program executes but still the obfuscator must control it, in particular inside loops. For instance, imagine that we execute

the program, increment  $x$  and continue the execution. Later, we happen to come back to a block we already executed, but with a different value of  $x$ . Since the test has not been modified, it will fail and we won't be able to find the next block to execute.

The solution is to actually number the blocks.  $s(i)$  denotes the value by which  $x$  is incremented when we travel from block  $i$  to block  $i + 1$ . Now suppose that we are currently in block  $i$  and that it contains a branch targetting an instruction in block  $j \neq i$ . If  $j > i$ , we increment  $x$  by  $\sum_{k=i}^{j-1} s(k)$ , otherwise we decrement  $x$  by  $\sum_{k=j}^{i-1} s(k)$ , before actually travelling through the branch. This method is all the more interesting as it makes  $x$  more unpredictable to a deobfuscator.

## 6 Probabilistically implementing $\tau$ -obfuscation and code protection

The main interest in considering  $\tau$ -obfuscation lies on the fact that code analysts and program obfuscators/deobfuscators have to face constraints that may greatly differ. To be more precise, let us consider  $\tau$ -obfuscation in a viral context. A piece of malware may fulfill its action in an amount of time that may be definitively unacceptable for an analysis program (namely an antivirus software). On a practical basis, which user would ever accept his antivirus to monopolize the computing resources during hours for analysis purposes<sup>7</sup>. On the contrary a malware can accept to take a long time to spread or attack. Consequently,  $\tau$ -obfuscation relies on two challenges:

- to protect the code against analysis: this is the purpose of code obfuscation itself.
- to make the deobfuscation process long enough to prevent, in the case of viruses for instance, an antivirus (or any other analysis program) to deobfuscate and identify the virus in less than a given time  $\tau$ .

The first objective can actually be achieved regardless of the second one. Indeed, in no way do we require that a virus should take less time than an antivirus to deobfuscate itself. So, rather than requiring from a virus to work in a very short time, why not imagine a malware that has no clue as to how it should deobfuscate itself? Then the virus would have to try several methods in order to do so and it might need two hours to eventually deobfuscate its code. In such a case, an antivirus would require the same time to deobfuscate the virus. But the main difference lies in the inability of an antivirus to take such a time to analyse any suspicious file.

Let us mention the fact that Zuo and Zhou [ZZ05] presented new results on time complexity of computer viruses (virus running time, virus detection procedure). Their main results are:

- For any type of computer viruses, there exists a computer virus  $v$  whose infecting procedure has arbitrarily large time complexity.
- For any type of computer viruses, there is a virus  $v$  such that any implementation of  $v$  can have arbitrarily large time complexity in its infection procedure.

Our approach as well as the concept of  $\tau$ -obfuscation directly relates to their results.

---

<sup>7</sup>Note that the most common protection against malware is “*on-access*” detection: for instance, before a file is executed or an e-mail attachment is opened. Though a user would not care whether the antivirus spends one minute or more to analyze a given document, he would certainly not accept this analysis to delay or differ the access to any resource (document, e-mail attachment, running an executable...).

## 6.1 A unified approach of code protection

Since obfuscation is only one possible technique that may be used to protect a code against analysis, let us make our approach more general and extend it to any other code protection technique: light code armoring<sup>8</sup>, polymorphism, metamorphism... Our goal is to define a unified perspective for code protection.

Let us consider a set  $\mathcal{S}$  of code protection methods (obfuscation, encryption, light code armoring, polymorphic/metamorphic rewriting techniques...) which may or not be conditioned by some secret information (e.g a key) to work. Thus, for any  $s \in \mathcal{S}$  and for any program  $P$ ,  $s(P)$  is a code that cannot be analyzed except with a sufficient amount of time  $\tau$ . Let us consider now the set  $\mathcal{S}'$  of methods which are defined as follows:  $\forall s \in \mathcal{S}, \exists s' \in \mathcal{S}'$  such that  $(s' \circ s)(P) \equiv P$ , where  $\circ$  denoted the (non necessarily commutative) functional composition<sup>9</sup>. In other words,  $\mathcal{S}'$  is the set of deprotection methods with respect to  $\mathcal{S}$ . It is important to note that  $|\mathcal{S}|$  and  $|\mathcal{S}'|$  may be not equal. Since we consider reversible code protection techniques, we assume that  $|\mathcal{S}| \leq |\mathcal{S}'|$ . Thus, for some  $s \in \mathcal{S}$ , there may exist  $\mathcal{S}'' \subset \mathcal{S}'$  such that  $\forall s'' \in \mathcal{S}'', (s'' \circ s)(P) \equiv P$ . Let us consider the following notation:

$$\mathcal{S}'_s = \{s' \in \mathcal{S}' | (s' \circ s)(P) \equiv P\}.$$

Let us now consider a given malware program  $P$  that embeds a code protection routine  $\mathcal{C}$ . This routine chooses at random a method  $s \in \mathcal{S}$  and compute  $s(P)$  which thus is a protected version of  $P$ . This formalism enables to describe many polymorphic techniques as the case where  $P \cap s(P) = \mathcal{C}$  whereas metamorphism is the case in which we have  $P \cap s(P) = \emptyset$ . Whenever the code  $s(P)$  runs, it deprotects itself by iteratively or randomly applying  $s' \in \mathcal{S}'$  to  $s(P)$ .

When considering a given  $s \in \mathcal{S}$ , the deprotection complexity obviously depends on the probability  $p_{s'}$  to find a  $s' \in \mathcal{S}'$  such that  $(s' \circ s)(P) \equiv P$ . Then, we have

$$p_{s'} = \frac{|\mathcal{S}'_s|}{|\mathcal{S}'|},$$

and the search for a deprotection method with respect to a given  $s \in \mathcal{S}$  has complexity  $\mathcal{O}(\frac{1}{p_{s'}})$ . Of course, this complexity and the corresponding probability hold under the assumption that the deprotection methods are uniformly and identically distributed over the set  $\mathcal{S}'$ .

Since we do not make any additional assumption about the elements in  $\mathcal{S}$ , this approach does not make any difference between light code armoring, polymorphic/metamorphic techniques, encryption techniques... Protection methods are considered instead. This unified view enables to focus only on the mathematical properties or structures the sets  $\mathcal{S}$  and  $\mathcal{S}'$  should exhibit rather than on the inherent nature of their respective elements (the protection methods).

From a practical point of view, an easy way of implementing such a method would be to choose a random protection method among a (possibly infinite) set of methods. To deprotect a protected code (e.g a virus), one would basically need to successively or randomly test all inverses of those methods. Let us now describe two implementations of code protection based on this approach. We will assume that only a (critical) part of the code may be protected. From a general point of view, the validation of the deprotection can be done in many ways. The most obvious and easy way to implement this, is to consider a hash function  $H$  [Fil05]. Whenever a deprotection method  $s'$  is applied to  $s(S)$ , the value  $H((s' \circ s)(P))$  is computed and compared to a validation data stored somewhere in the code.

<sup>8</sup>In this case the purpose is not to forbid code analysis like in [Fil05] but to delay automated code analysis (e.g. by antivirus).

<sup>9</sup>For some particular code protection techniques, we may have  $(s' \circ s)(P) = P$  (e.g. encryption)

## 6.2 Probabilistic encryption

Let us imagine that we want to encrypt some part of the code. If we do not care about the time we need to decrypt it but if we only require that any analyst (or any analysis software like an antivirus) cannot decrypt it in less than a given amount of time, then we could choose a random key of a given entropy  $n$ . When needing to decipher the code, we have to test any key of size  $n$  and so will do the analyst. Except that if the analyst has a great deal of data to process, this will be unachievable for him at least in a reasonable amount of time. In this general case, the average complexity is  $\mathcal{O}(2^n)$ . It is essential to note that the encryption algorithm must be carefully chosen so that no attack enables to recover the secret key more efficiently than with an exhaustive search. A good choice would be to consider a block cipher like GOST [28180]. Since the entropy key is 256 bits, a number of key bits may be fixed to a constant value in order to work with shorter keys. Let us stress on a very important point: **the aim is not to make the code unbreakable but to delay any automated cryptanalysis.**

We can achieve probabilistic encryption in several ways, among many others.

### 6.2.1 A first probabilistic scenario

As a first approach, we can consider choosing a random key for encryption then successively testing each possible key. If we suppose that testing a key takes one microsecond ( $10^{-6}$ ), then we will need at most slightly more than one hour to guess a 32-bit key. No antivirus can take so much a long time to analyze a malware. This solution is not optimal since some constraints have to be put on the size of the key that cannot have a too large entropy.

So we could also store a random permutation of the chosen key. However, once again, the set of possible keys would be too large (though this would depend on the presence of similar bits in the key): by restricting to a specific set of permutations, we can reduce the number of possible keys. We will discuss this in the upcoming section.

Another solution would be to hide the secret key in the encrypted data. Once the encrypted data is generated, the key is split into different chunks which are scattered all over the encrypted data. To do so, we can select random periods for insertion: given a first period  $p$ , we insert those parts every  $p$  bytes up to the end, then, given a second period, we continue inserting those chunks, and so on. Then, when looking for the key, we will have to resolve these periods, by random guessing. The number of periods is unknown, though it is limited by the size of the key. Thus, unless it is fixed, we need to know the size of the key before looking for those periods: this size should be stored in clear. The interest of such a method is to be free of any constraints on the key itself. Besides, the encrypted code itself is hidden inside the program, since the key is scattered over it and must be retrieved before being able to extract the encrypted code. The set of possible periods can also be controlled by controlling the extremes of the random generator.

Yet this method does not conceal perfectly the presence of the virus — though an antivirus will not be able to identify it. The presence of a great amount of encrypted code in the program will make the file look suspicious: this gives a clue to an antivirus as to where to look more specifically for malwares. This issue has been extensively addressed in [Fil06, Chap. 2, 6 and 7] and a number of solutions have been designed to efficiently fool entropy-based detection.

### 6.2.2 A more sophisticated approach

In this second approach, the part of the code to protect is encrypted by a public key method. A private/public key pair  $(K_{pr}, K_{pu})$  is randomly generated before protecting the code. After that, the private key  $K_{pr}$  is discarded. The public key  $K_{pu}$  is stored inside the deprotection routine. To deprotect the code, the private key  $K_{pr}$  has to be guessed by the deprotection routine. For

that purpose, instead of trying any possible  $K_{pr}$  – which would either require a too huge amount of time or limit the private key size more than desirable – this routine recovers it directly from  $K_{pu}$ . Let us consider the following example.

- We consider probabilistic public-key encryption. The main reason to choose this kind of encryption comes from the fact that it is semantically secure [GM84, MRS88]. In other words, it represents a polynomially bounded version of perfect secrecy as defined by Shannon [Sha49]. Probabilistic encryption is far more secure than its deterministic counterparts (RSA encryption as an example). It utilizes randomness to reach a provable and strong level of security. A passive adversary with polynomially bounded computational resources – this is obviously the case for any automated analysis and detection software – cannot learn anything about the plaintext from the ciphertext. Moreover it is then conceivable that there exist semantically secure encryption systems where the keys are much shorter than the message itself. In our case, we will use the Goldwasser-Micali probabilistic encryption [GM84] which is semantically secure assuming the intractability of the quadratic residuosity problem.
- We will not describe this probabilistic encryption scheme in detail but just describe the key generation part. Before any code protection step, the protection routine has to generate a private/public key pair  $(K_{pr}, K_{pu})$ . The general algorithm is:
  1. Select two large random, distinct primes  $p$  and  $q$ , each roughly the same size (by means of the Miller-Rabin algorithm [MvOV97, p. 146]).
  2. Compute  $n = pq$ .
  3. Select a  $y \in \mathbb{Z}_n$  such that  $y$  is a quadratic non-residue modulo  $n$  and the Jacobi symbol  $\left(\frac{y}{n}\right) = 1$ .
  4. The public key  $K_{pu}$  is the pair  $(n, y)$  and the private key  $K_{pr}$  is  $(p, q)$ .
- The protection routine then protects the code by probabilistically encrypting it with the key  $K_{pu}$ . Then, the pair  $(p, q)$  is discarded.

The security of the Goldwasser-Micali probabilistic encryption scheme is based on the quadratic residuosity problem (*QRP*) intractability. *QRP* is itself as hard as the problem of factoring integers. Consequently, according to what has been previously considered, the deprotection routine has to factor the number  $n$ . A few efficient algorithms are known to factor integers [Mor02, Len00], and the best running time is in  $\exp(c \log(\alpha) \cdot n \cdot (\log(\log(n)))^{1-\alpha}) = L_n[\alpha, c]$  where  $\alpha$  and  $c$  are some constants (for the best methods  $\alpha = \frac{1}{3}$  and  $1.5 < c < 2$ ). However, most of these efficient methods require a huge amount of memory, far more than any commercial antivirus may consider. Moreover, some mathematical and implementation constraints must be considered in order to achieve an optimal running time. That is why, for code deprotection purposes, most of these methods cannot be considered. The best trade-off can be obtained with the *quadratic sieve* (QS) methods [Pom84]. Let us note  $L_n[\alpha] = \exp(\sqrt{\log n \log \log n})^{\alpha + \mathcal{O}(1)}$ . Then, the QS running time complexity when factoring an integer  $n$  is  $L_n[1]$  and the memory complexity is  $L_n[\frac{1}{2}]$ . Thus, the choice of the public key  $K_{pu}$  has a direct impact on the deprotection method. Simulations have shown that a 50-digit key could be factored in nearly 30 minutes requiring less than 100 Kb of memory.

*Remark 1.-* Public key encryption can also be used to encrypt a symmetric secret key  $K$  only (in other words, a session key as in PGP encryption software). Referring to Section 6.2.1, the interest lies in the fact that the encrypted key  $E_{K_{pu}}(K)$  hides the exact entropy of  $K$ .



### 6.3 Data transformation

Let us consider that we have at our disposal a set of methods that perform a transformation of the virus data (incl. code or raw data). Obfuscating the code would mean applying one of these transformations to the virus data, whereas deobfuscating it would mean finding the inverse transformation. Let  $\mathcal{S}$  be this set of transformations. Among several approaches, let us consider that  $\mathcal{S}$  has a group structure with regard to the law of composition. Then for any transformation  $\pi \in \mathcal{S}$ , its inverse  $\pi^{-1}$  is also in  $\mathcal{S}$ , as well as any composition of transformations of  $\mathcal{S}$ .

The symmetric group  $\sigma_n$  – the group of permutations of  $[1..n]$  – is a convenient example for this set of transformations. The main interest in considering data transformation by code permutation lies in the fact that encrypted content has a very specific statistical profile whereas permuted data have not. While a detection software will trigger an alarm whenever the code is encrypted<sup>10</sup>, it will not if the code is only a permutation of itself.

Obfuscating the virus data would be achieved by permuting the bytes of these data. We will have to consider a *cyclic subgroup*  $G$  of  $\sigma_n$ , in order to restrict its size ( $|\sigma_n| = n!$ ). This group has several interesting properties. In particular, for a given transformation  $\pi \in \sigma_n$ , its inverse is  $\pi^k$ , for a  $k \in [1..n]$ . Thus the protection routine just picks up a permutation  $\pi$  at random whereas the deprotection procedure picks up a random permutation  $\pi'$  and exhaustively searches for the right  $k'$  such that  $(\pi')^{k'} \circ \pi = Id$ . In this general setting, since the deprotection complexity is  $\mathcal{O}(|G|)$ , we have to choose a large value of  $n$  in order to make the deprotection time long enough. From a practical point of view, the permutation  $\pi$  may be chosen as the generator of a cyclic subgroup of  $\sigma_n$ . Finding the inverse permutation is consequently easy to implement by means of a simple loop which ranges from 1 to the order of the subgroup. Moreover, the subgroup order will be chosen as a prime number in order to maximize the number of subgroup generators.

In the previous case, since any permutation has a unique inverse, the set  $\mathcal{S}'$  is a singleton. But we may desire for a given code protection method  $s \in \mathcal{S}$  to have  $|\mathcal{S}'_s| > 1$ . This would offer more flexibility in implementing code protection/deprotection. One solution – among others – is to consider a fixed ordered set  $\mathcal{F}$  of (possibly non-contiguous) bytes  $\{b_{i_1}, b_{i_2}, \dots, b_{i_k}\}$  and a set of code locations  $\{i_1, i_2, \dots, i_k\}$ . Whenever deprotection of a program  $\mathcal{P}$  yields

$$\forall i \in \{i_1, i_2, \dots, i_k\}, \quad \mathcal{P}[i] = b_i,$$

then the code is considered as deprotected<sup>11</sup>. In this case, the set  $\mathcal{F}$  describes the critical part of the code that must be permanently protected (by randomly permuting it) while the other (permuted) part of the code may be dummy bytes or dead code. Let us consider the following permutation of a 9-byte program:  $\pi = (384729156)$ . If  $\mathcal{F}$  contains the original bytes of the program whose location indices are  $\{2, 5, 8\}$  then  $\mathcal{S}' = \{\pi^3, \pi^6, \pi^9\}$ .

From a general point of view, it is possible to partition the code into  $k$  sets. We choose  $k - j$  sets to describe the critical part of the code to protect while the  $j$  remaining sets are the non critical part of the code or are dummy/dead code. We then consider  $k$  different cyclic permutation groups, each of them acting over one of the  $k$  sets. This latter approach enables to consider permutation subgroups of far larger size. Indeed, the order of a cyclic subgroup has the same magnitude as the permutation length. In other words, when considering a single cyclic permutation group of order  $n$ , any antivirus will have to try at most  $\mathcal{O}(n)$  permutations to find its inverse. On the contrary, when considering  $k$  different cyclic subgroups, each of them of order  $n_i$ ,  $i = 0, \dots, k - 1$ , the complexity rise to  $\mathcal{O}(\text{lcm}(n_0, n_2, \dots, n_{k-1}))$  with a maximal value if the different  $n_i$  are coprime. Of course, an efficient implementation will consider both random value

<sup>10</sup>Some Internet providers do not deliver e-mail which contains encrypted attachment, as an example.

<sup>11</sup>Let us recall that the validation would of course not compare the raw data but for instance, test the value of its hash.

of  $k$  and random (coprime)  $n_i$ . Moreover, in order to forbid the analyst to guess what is the critical portion really is, and thus to restrict the search of the inverse permutation to this critical part, an efficient implementation of this protection scheme will prevent this in the following way: the protected, critical part – let us denote  $\sigma(\mathcal{C})$  – contains the length over which to apply the validation hash. If the inverse permutation is not correct, then  $(\sigma' \circ \sigma)(\mathcal{C})$  will contain a random value of this length thus fooling the analyst: this length may be larger or shorter than the exact required length, thus any restriction with regards to the inverse permutation length is impossible.

## 7 Conclusion

In this paper, we have studied how to practically obfuscate executable code. Following Barak & al results [BGI<sup>+</sup>01], we have proved that with a slightly different formalism, obfuscation still does not exist. But if any obfuscated code is bound to be successfully analyzed by a human operator, automated programs (e.g. antivirus software or Intrusion Detection Systems (IDS)) with constant ( $\mathcal{O}(1)$ ) bounded resources are bound to fail. This is the main perspective of our work. It is thus conceivable that efficient practical obfuscation techniques will put check on any detection software in the future, if not already the case.

The practical obfuscation techniques we have presented in Section 5 and 6 have been essentially considered in the case of malware context and under some very particular conditions. But code obfuscation techniques may have a huge number of positive applications (e.g. copyright enforcement, digital right management...). The authors are considering as an open problem whether these techniques may be used for such applications.

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