Reverse Engineering MAC: A Non-Cooperative Game Model

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Summary

Reverse engineering:

Given the solution, what is the problem?

Know what works, what doesn't, why it works, how to improve.

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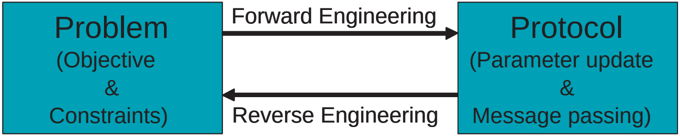
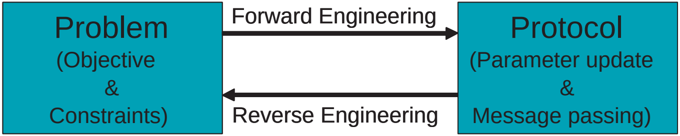
Provide the missing piece (on MAC) for rigorous mathematical understanding of existing layers 2-4 protocols

Reverse Engineering



Reverse Engineering



- Related works:
  - Layer 4: TCP/AQM [Kelly-Maulloo-Tan98, Low03, Kunniyur-Srikant03, ...] NUM
  - Layer 3: BGP [Griffin-Shepherd-Wilfong02] SPP
  - Layer 2: MAC (contention avoidance in random access) [This Talk]

Review: TCP/AQM

Network Utility Maximization Problem

$$\begin{aligned} \text{maximize} \quad & \sum_s U_s(x_s) \\ \text{subject to} \quad & \sum_{s:l \in L(s)} x_s \leq c_l, \quad \forall l, \\ & \mathbf{x}^{min} \preceq x \preceq \mathbf{x}^{max}. \end{aligned}$$

- $U_s(x_s)$: utility of each user depends on its own data rate
- Adequate feedback from the network

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- $U_s(x_s)$: utility of each user depends on its own data rate
- Adequate feedback from the network
- Reverse engineering provides
  - Better understanding: existence, uniqueness, optimality and stability, counter-intuitive behaviors
  - Systematic design: scalable price signal, control laws with better stability properties

MAC Reverse Engineering

- Utility of each link depends on transmission probabilities of all links
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Reverse engineer to non-cooperative game

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Reverse engineer to non-cooperative game

- Questions:
  - What are users' utility functions?
  - What does the MAC protocol do for the game?
  - What are the properties of the Nash Equilibrium (result of game)?

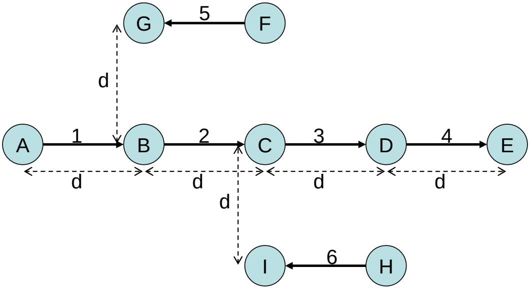
Different Work

Game to MAC:

- MacKenzie, Wicker 2003
- Jin, Kesidis 2004
- Altman et. al. 2005
- Yuen, Marbach 2005
- Wang, Krunz, Younis 2006

- This is different: Reverse engineering
- Discover, not impose, utility and game

Sample Network



Persistence Probabilistic Model of Protocol

- Protocol parameters:
  - $p_l^{\max}$: Maximum persistent probability (politeness)
  - $p_l^{\min}$: Minimum persistent probability
  - $\beta_l \in (0, 1)$: Backoff multiplier

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- Protocol description: link l transmits with a probability p_l
  - If success (no collision), update $p_l = p_l^{\max}$
  - If failure (collision), update $p_l = \max\{p_l^{\min}, \beta_l p_l\}$, where $0 < \beta_l < 1$

Persistence Probability Update

Persistence Probability Stochastic Update

$$p_l(t+1) = \max\{p_l^{min}, p_l^{\max}\mathbf{1}_{\{T_l(t)=1\}}\mathbf{1}_{\{C_l(t)=0\}} + \beta_l p_l(t)\mathbf{1}_{\{T_l(t)=1\}}\mathbf{1}_{\{C_l(t)=1\}} + p_l(t)\mathbf{1}_{\{T_l(t)=0\}}\}$$

- $T_l(t)$: link l transmits at time slot t

$$\mathrm{Prob}\{T_l(t) = 1|\mathbf{p}(t)\} = p_l(t)$$

- $C_l(t)$: at least one link that can cause collision to link l transmits at t

$$\mathrm{Prob}\{C_l(t) = 1|\mathbf{p}(t)\} = 1 - \prod_{n \in L_{to}(l)} (1 - p_n(t))$$

Deterministic Approximation

Persistence Probability Update: Deterministic Approximation

$$
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p_l(t+1) \quad = \quad & \max\{p_l^{min}, p_l^{max} p_l(t) \prod_{n \in L_{to}(l)} (1 - p_n(t)) \\
& + \beta_l p_l(t) p_l(t) \left(1 - \prod_{n \in L_{to}(l)} (1 - p_n(t))\right) \\
& + p_l(t)(1 - p_l(t))\},
\end{aligned}
$$

- Links are playing a game
- Each link l tries to maximize its utility U_l based on other links' current transmission probabilities

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- Links are playing a game
- Each link l tries to maximize its utility U_l based on other links' current transmission probabilities
- **Key question**: what is the game model?

MAC Game

Definition

A MAC game is $[E, \times_{l \in E} A_l, \{U_l\}_{l \in E}]$

- E: set of players (links)
- $A_l = \{p_l | p_l^{\min} \leq p_l \leq p_l^{\max}\}$: action set of link l
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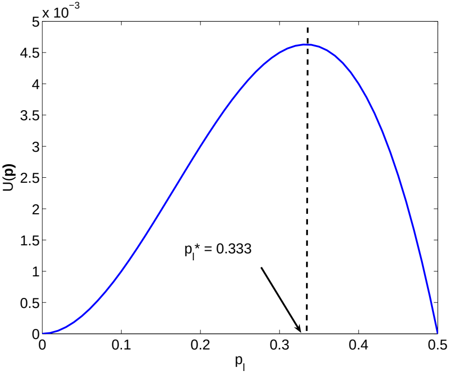
Theorem

Utility function turns out to be expected net reward:

$$U_l(\mathbf{p}) = R(p_l)S(\mathbf{p}) - C(p_l)F(\mathbf{p})$$

where $R(p_l)$ is reward for transmission success,
$S(\mathbf{p})$ is probability of transmission success,
$C(p_l)$ is cost for transmission failure,
$F(\mathbf{p})$ is probability of transmission failure.



Dependence of a utility function on its own persistence probability ($\beta_l = 0.5$, $p_l^{max} = 0.5$, and $\prod_{n \in L_{to}(l)}(1 - p_n) = 0.5$)

Interpretation of MAC protocol: a stochastic subgradient algorithm

- Is it a gradient-based maximization of $U_l(\mathbf{p})$ over p_l?
  - No, that requires explicit message passing among links

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- MAC maximizes U_l using stochastic subgradient ascent method (using only local information on success and collision):

$$p_l(t+1) = \max\{p_l^{min}, p_l(t) + v_l(t)\}$$

where

$$\mathrm{E}\{v_l(t)|\mathbf{p}(t)\} = \frac{\partial U_l(\mathbf{p})}{\partial p_l}|_{\mathbf{p}=\mathbf{p}(t)}$$

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- Assume all links have the same $p^{\max} < 1$ and $p^{\min} = 0$

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Theorem

There always exits a Nash equilibrium in the MAC game, which can be characterized by

$$p_l^* = \frac{p^{max} \prod_{n \in L_{to}(l)} (1 - p_n^*)}{1 - \beta_l (1 - \prod_{n \in L_{to}(l)} (1 - p_n^*))}, \ \forall l.$$

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- Proof: Fixed point theorem in the compact strategy interval.
- The Nash equilibrium may not be unique in general.

Uniqueness and Convergence of Nash Equilibrium

- Define the best response function as

$$p_l^*(t+1) = \arg \max_{p_l^{\min} \leq p_l \leq p_l^{\max}} U_l(p_l, p_{-l}^*(t))$$

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Define maximum interference degree as $K = \max_l |L_{to}(l)|$, *then if*

$$\frac{p^{max} K}{4\beta(1 - p^{max})} < 1$$

- *The Nash equilibrium is unique*
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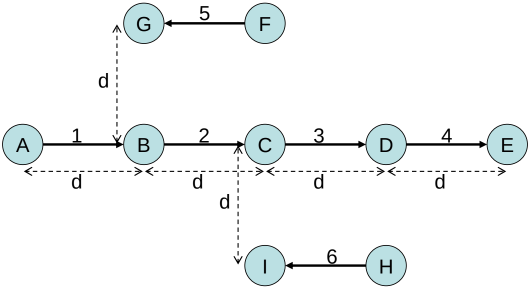
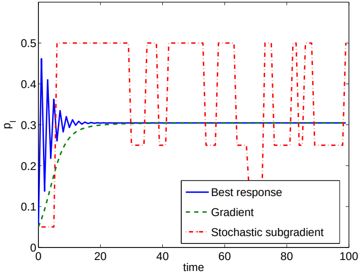
- Proof: Properly bounding the matrix norm of the Jacobian. Show it is a contraction mapping.
- How polite is necessary? Critical value: $p_c^{\max}$

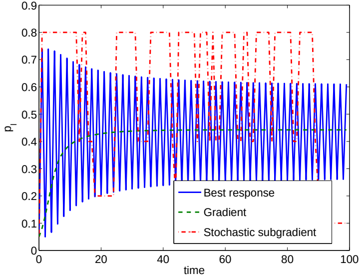
Network Topology



A network with Six Links

Convergence





$p^{\max} = 0.5$ $\qquad\qquad$ $p^{\max} = 0.8$

Comparison of trajectories of $p_l(t)$ in the network

Summary

- **Topic**: reverse engineering of MAC protocol
- **Key idea**: a non-cooperative game model
- **Results**:
  - Utility function discovered: expected net reward
  - Current MAC algorithm corresponds to stochastic subgradient update
  - NE always exists. It is unique and stable if the protocol is polite enough and backoff smooth enough
- **Implications**:
  - Reverse engineering leads to deeper understanding of existing protocols
  - Insights are helpful for better forward engineering